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SOME CONSIDERATIONS ON MEASURING THE NEWTONIAN GRAVITATIONAL CONSTANT G IN AN ORBITING LABORATORY

Prepared by:	Stephen D. Baker
Academic Rank:	Professor
University and Department:	Rice University Department of Physics
NASA/MSFC:	
Laboratory:	Space Science
Division:	Astrophysics
Branch:	High Energy Astrophysics
MSFC Colleague:	C. A. Meegan
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ABSTRACT

We have identified no fundamental reason for rejecting the notion of measuring the Newtonian gravitational constant G by observing an artificial binary in a near-earth orbiting laboratory.

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INTRODUCTION

Of the fundamental contants of nature, the Newtonian gravitational constant G is by far the least well known (only about one part in ten thousand, or worse). The use of artificial binaries to determine this constant has been regularly proposed since it became thinkable to have orbiting laboratories. A regular objection to this method has been that, in near earth orbit, the gravitational field of the Earth is changing so rapidly with distance from the earth that "tidal forces" would make the orbits of artificial binaries unstable. This is not the case, however; certain retrograde orbits are indeed stable and could be used to sample the gravitational attraction between two balls for a long time, thus permitting the determination of G.

OBJECTIVES

The objectives of the summer's activity were to become further familiar with the literature relevant to the problem at hand and to draw any further conlusions concerning the feasibility of measuring G in a near earth orbiting laboratory. A secondary objective was to complete a manuscript on this topic, with Adam Falk, begun last summer, for submission to the American Journal of Physics.

REPORT

A common suggestion for measuring the Newtonian gravitational constant in an orbiting laboratory is simply to put two balls in orbit around each other and measure the resulting orbital elements and period of the motion of such an artificial binary, thereby determining G. However, the radial variation with distance of the gravitational field of the earth is so large that "tidal forces" on the balls in near-earth orbit can be several times greater than the gravitational attraction between them. The presence of these relatively strong tidal forces close to the earth has led some writers to assume that two objects will not stably orbit about each other and that this method of measuring G is impossible, or at least impractical. Here is what they say:

Farinella, Milani, and Nobili (1980)

"This means that the physical limits to the density of [the binaries] imply that in a low Earth orbit no stable motion of the test mass around the primary is possible."

Avron and Livio (1986)

"Unfortunately, such an experiment is not possible with the Space Shuttle, the main reason being its low altitude."

Hills (1986)

"If the semimajor axis of the binary is sufficiently large, the tidal field would force its dissociation."

However, Michel Henon in a beautiful series of papers has explored the regions of long-term stability of many classes of orbits of point particles which are moving under their mutual gravitation in a frame which is orbiting about a large third body. He finds that certain retrograde orbits are stable and states (in 1970) that:

"Contrary to the usual [and, evidently, persistent] belief, there is no limiting [orbital] radius for the satellites."

Henon provides a very complete description of what is known as Hill's Problem, where the motion of one of two two equal mass balls is described relative to an orbiting frame of reference in terms of dimensionless coordinates:

$$\ddot{z} = -2\dot{x}$$

$$\ddot{x} = 2\dot{z} + 3x$$

$$\ddot{y} = -y$$

where the time unit is chosen to be $T/2\pi$, where T is the orbital period of the spacecraft and the length unit is chosen to be $(m/4M)^{1/3}$ A, where M is the mass of the earth and A is the radius of the orbit of the spacecraft.

Since one must use balls of finite density (and therefore finite size) we may investigate the tolerances on the inital conditions of the orbit so that the balls collide neither with each other nor with the walls of the spacecraft. In Figure 1 we show the "launch window" for a 10 kg, 5 cm radius, ball started out on the neutral line at z = 15 cm with the velocity components shown on the axes of the figure. The other identical ball is launched symmetrically to maintain the center of mass at the origin. Trajectories which result in the balls' striking each other or the enclosure of the spacecraft are represented by diamonds or crosses, respectively. For this example, the enclosure is taken to be a cube of edge length 110 cm. Those trajectories which survive for 25 orbital periods about he earth are unmarked. Launching from points other than on the neutral line (the z axis) is, of course, possible, and we find that the allowable ranges on the initial velocities are very similar to those in Figure 1.

While the tidal forces can be significantly reduced by going to a laboratory orbiting the earth at a large radius, it turns out that one does not gain much in terms of allowable ranges of initial velocities. We conclude therefore that tidal forces do not pose any particular problem in using an artificial binary in an experiment to determine G.

Are there other fundamental processes which might make it impossible to carry out such an experiment? We can consider several items:

1. Gradients due to components of the spacecraft. The spacecraft in which the experiment would be performed should be constructed to provide a minimal gradient at the position of the orbiting balls, but it is not difficult to do this. Since the experiment is in free fall, it is the gradient of the gravitational field which affects the relative motion of the balls and not the strength of the field itself. For example, the gradient due to a 100 kg mass a distance 5 m from the center of the experiment gives a gradient only 4.0E-5 times that of the earth.

2. Fluctuations in the gradient due to orbital eccentricity and non-sphericity of the Earth. The oblateness of the earth would create a variation of about 1% in the earth's gradient if the experiment orbits in a circular polar orbit, and less in an equatorial orbit. Eccentricity of the orbit would also give rise to variations in the gradient (about 1% for variations of 20 km in altitude). These effects are accurately calculable, however, and could be accurately compensated for.

3. Electrical charging of the balls. This should not present insurmountable problems, either. For example, to keep the electrostatic force a million times smaller than the gravitational force between the balls, used in the example shown in Figure 1, the charge on each ball would have to be less than about one picocoulomb, not impossibly small. Larger balls would permit larger charges.

CONCLUSION

We conclude therefore, that there is still no fundamental reason to believe that an experiment to use an artificial binary in a near-earth orbiting laboratory is an unreasonable way to obtain a very good determination of the Newtonian gravitational constant G.

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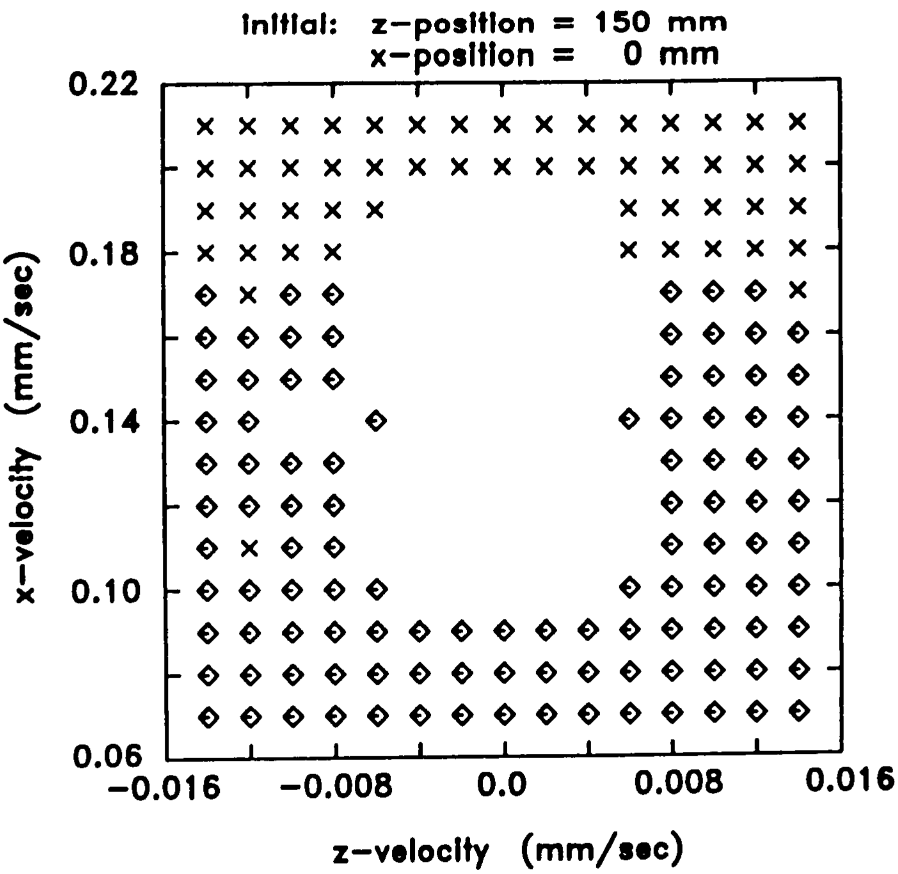

Figure 1